# Shear fabrics of naturally deformed galena 

H. Siemes and H. J. Spangenberg<br>Institut für Mineralogie und Lagerstättenlehre, Rheinisch-Westfälische Technische Hochschule, D 5100 Aachen, Germany

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#### Abstract

The analysis of X-ray pole figure data of laminated galena (Bleischweif) from Braubach, Germany reveals two components of preferred orientation. Both components fit to a simple model related to a simple shear deformation process. The predominant slip system $\{001\}<110>$ tends to align with the $\{001\}$-plane parallel to the shear plane and with the $<110>$-direction parallel to the shear direction. The first component of preferred orientation is close to a single crystal orientation with $\{001\}<110>$ parallel to the shear plane. The second component forms an incomplete girdle with $<110>$ close to the shear direction. In all cases the resolved shear stress in the main gliding system achieves high relative values between 0.6 and 1.0 .


## INTRODUCTION

Laminated galena (Bleischweif) is supposed to have originated'from a simple shear deformation process at temperatures below $200^{\circ} \mathrm{C}$ (McClay \& Atkinson 1977, McClay 1978, 1980, Siemes \& Schachner-Korn 1965). The flow plane is assumed to be parallel to the lamination and the flow direction parallel to the lineation. These assumptions are supported by microfabric studies (McClay 1978, 1980) by X-ray fabric measurements (Siemes \& Schachner-Korn 1965, McClay 1978, 1980) and by analyses as well as simulations of preferred orientation (Siemes \& Schachner-Korn 1965, McClay 1978, 1980). The analysis of preferred orientation of a laminated galena from the Rosenberg Mine, Königsstieler Gangzug, Braubach, West Germany by means of the orientation distribution function (ODF) is presented here as an example.

## GRAIN FABRIC

The galena sample from Braubach shows a well developed lamination or foliation (Siemes 1977, fig. 8) which is recognizable by the visible preferred orientation of the $\{001\}$-cleavage. The sample was cut parallel to three planes perpendicular to each other, one plane approximately parallel to the foliation ( $S$ ), the others perpendicular to the foliation and approximately perpendicular and parallel to the lineation ( $L$ ) respectively. Polished sections of these three planes were etched (Bebrick \& Scanlon 1957). The mean diameters of the polygonal grains in the three sections are $50 \mu \mathrm{~m}$ $(\| S), 37 \mu \mathrm{~m}(\perp L)$, and $39 \mu \mathrm{~m}(\perp \mathrm{~S} \| L)$. This implies that the grains have an ellipsoidal shape with a ratio of 1:0.74:0.78. The long and intermediate axes lie in the $S$-plane and the longest axis is oriented parallel to the lineation. Besides this the numerous boulangerite inclusions, with mean dimensions of $35 \cdot 10 \cdot 10 \mu \mathrm{~m}$, have their long axes aligned parallel to the lineation.

## PREFERRED ORIENTATION

The preferred orientation of the galena was measured in all three planes in the back-reflection mode by means of an X-ray pole-figure-analysis system which has been designed by Lücke and is manufactured by the Siemens Company (Kobbe \& Schuon 1973). The data of the (200)-, (111)-, (220)- and (422)-reflections were combined to form complete pole figures and were projected parallel to the plane of lamination by means of a computer program (Siemes 1977). The data processing includes some adjustment of the pole figures in relation to the plane of lamination and the lineation. This adjustment differs slightly from that in a previous publication (Siemes 1977) and as a result there are small differences in the respective pole figures.

From the distribution of the maxima and minima areas one can derive an $\mathrm{E}-\mathrm{W}$ mirror plane that shows the monoclinic symmetry of the fabric (see Fig. 1a). Because of this symmetry all the pole figures were made symmetric across an E-W axis (see Fig. 1b). The symmetric pole figures were processed by a computer program written by Spangenberg (1977) in order to calculate the three-dimensional orientation distribution function (Bunge 1969, Bunge \& Wenk 1977).

The degree of expansion for the calculation was $L_{\text {max }}$ $=20$. The result is a series of $C$-coefficients and related error quantities $\overline{\Delta C}_{1}$. A detailed explanation of the procedure is given in Bunge (1969) and Bunge \& Wenk (1977). In order to check for errors and the resolution of the function, the mean values, $\bar{C}_{1}$ of the $C$-coefficients are plotted together with the error quantities $\overline{\Delta C}_{1}$ in Fig. 2. It can be seen from Fig. 2, that the errors are small and that the values for $\bar{C}_{1}$ and $\overline{\Delta C}_{1}$ converge rapidly. Therefore the selected degree of $L_{\text {max }}=20$ is sufficient for the calculations. The texture index is $J=1.63$, which is rather small and indicates a weak preferred orientation. From the $C$-coefficients the pole figures were recalculated. The agreement between the original symmetric pole figures and the recalculated pole figures is quite


Fig. 1. Equal area projection of the lower hemisphere of (220)-pole figures of laminated galena from Braubach, West Germany, lamination parallel to the plane of projection, $\mathrm{A}=$ lineation. The increment between the isolines is 0.2 and in multiples of the uniform density $(=1.0)$.
(a) Complete poie figure as projected after combining the data from the three sections perpendicular to each other. (b) Symmetric pole figure, averaged across the E-W mirror plane.
good (Fig. 3).
The orientation distribution function was calculated from the same coefficients. This is a density function in three-dimensional Euler space. The axes of this space specify the orientation of the crystal-coordinate system with respect to the specimen-coordinate system by means of the Eulerian angles $\varphi_{1}, \Phi, \varphi_{2}$ as defined by Bunge (1969). Figure 4 shows 9 sections perpendicular to the $\varphi_{2}$-axis through this space for the laminated galena from Braubach.

In each section two maximum areas can be distinguished, that means that there are two components of preferred orientation. The first one consists of 2 continuous tubes and is labelled with squares. The density is varying along the tube between 3.4 and 4.5. The second


Fig. 2. The mean $C$-coefficients $\bar{C}_{1}$ and the error quantities $\overline{\Delta C}_{1}$ for the least squares solution for $L_{\text {max }}=20$.
one consists of cigar shaped areas of high concentrations. In the section $\varphi_{2}=50^{\circ}$ the cigar vanishes near the position $\varphi_{1}=60^{\circ}, \Phi=135^{\circ}$ and a new cigar arises in the section $\varphi_{2}=40^{\circ}$ near the position $\varphi_{1}=55^{\circ}, \Phi=65^{\circ}$. This second component of preferred orientation is labelled with dots. Along its axis the density varies between 2.0 and 4.5.

## INTERPRETATION OF PREFERRED ORIENTATION

In order to recognize the meaning of both distributions in the Euler space, the coordinates of the maxima in 18 sections with a distance of $5^{\circ}$ along the $\varphi_{2}$-axis were read and the cubic galena crystal was rotated in the appropriate positions by means of a computer program. The $\{001\}$-, $\{111\}$ - and $\{110\}$-poles were plotted separately in projections parallel to the lamination (Fig. 5).

The pole positions of the first preferred orientation component are again marked by squares, they represent a nearly single crystal orientation with $\{001\}$ parallel to the lamination and $<110\rangle$ parallel to the lineation. That means that the $\{001\}$-glide plane of galena is oriented parallel to the shear plane and the $<110\rangle$ glide direction parallel to the shear direction. This translation system is the predominant glide system between $300-400^{\circ} \mathrm{C}$ (McClay 1978).

The second component of preferred orientation (marked by dots in Fig. 5) produces incomplete girdles of poles. All crystals of this orientation distribution are closely aligned with one $<110>$-direction parallel to the lineation and parallel to the shear direction respectively. In the incomplete girdles the orientation of the crystallographic planes parallel to the shear plane varies from $\{001\}$ to $\{111\}$ (Fig. 7b).


Fig. 3. Comparison of the symmetric measured pole figures (upper half) and the pole figures recalculated from the $C$-coefficients (lower half). Lamination parallel to the plane of projection, $A=$ lineation. (a) (200). (b) (111). (c) (220).

The derived preferred orientations of the laminated galena from Braubach can be explained by a very simple model. During the shearing of the galena, glide occurs in several planes and one of the glide planes tends to align with the shear plane and in this glide plane one of the glide directions tends to align with the shear direction. This means, that the resolved shear stress in the glide direction achieves a high value. The relationship between the applied shear stress for a simple shear deformation and the resolved shear stress in the glide direction is given by following a formula (Gough et al. 1926):
$M=\tau / \tau_{t}=\sin \chi \cos \lambda \sin \psi_{T}+\cos \chi \sin \lambda \sin \psi_{t}$
where (see Fig. 6):
$\tau_{1}=$ resolved shear stress in the translation direction $t$;
$\tau=$ applied shear stress in the shear direction Sh ;
$\chi=$ angle between the normal to the shear plane $N_{s}$; and the normal to the translation plane $N_{T}$;
$\lambda=$ angle between the normal to the shear plane $N_{S}$ and the translation direction $t$;
$\psi_{T}=$ angle between $R$ (normal to the shear direction Sh in the shear plane $S$ ) and the projection of $N_{T}$ on $S$;
$\psi_{t}=$ angle between $R$ and the projection of the translation direction $t$ on $S$.
For the first orientation component the value of $M$ is close to 1.0 , i.e. the maximum value (Fig. 7a). For the second component which forms incomplete girdles, the value ranges from about 0.6 for the extreme position with $\{111\}$ nearly parallel to the shear plane (Fig. 7b) to to 1.0 with $\{001\}$ nearly parallel to the shear plane.

Preliminary simulations, of preferred orientations in galena, making use of the model of Taylor (1938) and Bishop \& Hill (1951) and the programs of Lister (1974) as given in McClay $(1978,1980)$ produce the same final positions as measured in the Braubach galena.

## CONCLUDING REMARKS

The above orientation distribution function for laminated galena needs to be supplemented by others for different samples, as it is known that different types of preferred orientation patterns exist (McClay 1978, Siemes \& Schachner-Korn 1965, Schachner-Korn $1947 / 49$ ). Further analyses are necessary before the orientation distribution function can be calculated without making assumptions about the specimen symmetry (see Bunge \& Wenk 1977). Also, to avoid errors due to the X-ray measurements from three mutually perpendicular planes, it would be better to use neutron diffraction (Kleinstück et al. 1976, Wagner et al. 1977) or to make use of the methods of Ruer \& Baro (1977, see p. 199), which can make use of incomplete pole figures from materials of any crystallographic system.

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Fig. 4. (continued).


Fig. 5. Plotted pole positions for two components of preferred orientation of the orientation distribution function of Fig. 4. $C=$ pole of lamination, $A=$ lineation, component $1=\boldsymbol{D}$, component $2=\boldsymbol{\theta}$. Equal area projections of the lower hemisphere: (a) $\{100\}$-poles; (b) $\{111\}$-poles; (c) $\{110\}$-poles.


Fig. 6. Geometry of the shear plan $S$ with the shear direction Sh and the translation glide plane $T$ with the glide direction $t$. The indicated angles $x, \lambda, \psi_{n}$ and $\psi_{t}$ are necessary to calculate the resolved shear stress $\boldsymbol{\tau}_{\text {, }}$ in the shear direction $\mathbf{S h}$.
orientation-distribution function and provided the $X$-ray equipment. The computer programs were run on the Cyber 175 of the Computer Center of the RWTH Aachen. The excellent specimen of laminated galena was supplied by Dieter Muller of Stolberg. Thanks are due to the staff of the Mineralogical Institute for their technical assistance.

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Fig. 7. Two single orientations recalculated according to the indicated Euler angles. $C=$ pole of lamination, $\boldsymbol{A}=$ lineation.
$\square=\{001\}, \Delta=\{111\}, O=\{011\}, X=$ glide plane, $\otimes=$ glide direction . Equal area projections of the lower hemisphere.
(a) orientation of one example of component 1 with \{001\} nearly parallel to the shear plane. Euler angles: $\varphi_{1}=20^{\circ}, \Phi=10^{\circ}, \varphi_{2}=25^{\circ}$, density of this orientation in the ODF: 3.7; relative shear stress $M=$ 0.98 . (b) Orientation of one example of component 2 with $\{111\}$ nearly parallel to the shear plane. Euler angles: $\varphi_{1}=60^{\circ}, \Phi=125^{\circ}, \varphi_{2}=35^{\circ}$; density of this orientation in the ODF: 2.2; relative shear stress

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M=0.62
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